Computing Probabilities for Variables that follow the Normal Distribution

(This is a reminder of material we discussed in class and is not sufficiently discussed in the textbook.)

We know that given a continous random variable $X$ it is

$$P(a \leq X \leq b) = \int_{a}^{b} f_{X}(x)dx$$

where $f_{X}$ is the distribution of $X$. So, the computation of the probability $P(a \leq X \leq b)$ reduces to computing an integral.

Unfortunately life is not so easy when $X$ follows the normal distribution, which is the well-known bell curve. The reason is that it is impossible to compute $\int_{a}^{b} \phi_{\mu,\sigma}(x)dx$ using the standard techniques of calculus, where $\phi_{\mu,\sigma}$ is the normal distribution with mean $\mu$ and standard deviation $\sigma$. The good news is that people have already computed and created tables for

$$P(0 \leq T \leq t)$$

for the case where $T$ follows the standard normal distribution, i.e., the normal distribution with $\mu = 0$ and $\sigma = 1$. This is the table that was handed out.

Now, let’s assume that you are asked to compute $P(a \leq X \leq b)$. The recipe is simple. First, transform the problem on $X$ into a problem on $T$.

$$P(a \leq X \leq b) = P\left(\frac{a-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{b-\mu}{\sigma}\right) = P\left(\frac{a-\mu}{\sigma} \leq T \leq \frac{b-\mu}{\sigma}\right) = P(a' \leq T \leq b')$$

Next comes a case analysis on $a'$ and $b'$:

1. If $a' = 0$ and $b' > 0$ then $P(0 \leq T \leq b')$ is given by the handed-out table.

2. If $a' < 0$ and $b' = 0$ then $P(a' \leq T \leq 0) = P(0 \leq T \leq -a')$, which is given by the handed-out table.

3. If $a' > 0$ and $b' > 0$ then $P(a' \leq T \leq b'') = P(0 \leq T \leq b') - P(0 \leq T \leq a')$ and both $P(0 \leq T \leq b')$ and $P(0 \leq T \leq a')$ are given by the handed-out table. The case for $a' < 0$ and $b' < 0$ is done in a similar fashion (try at home).

4. If $a' < 0$ and $b' > 0$ then $P(a' \leq T \leq b'') = P(0 \leq T \leq b') + P(0 \leq T \leq -a')$ and both $P(0 \leq T \leq b')$ and $P(0 \leq T \leq -a')$ are given by the handed-out table.