1. For these problems the student gets full credit if they get the answer correct, zero points otherwise.

1. B - the answer is $\frac{4}{5}$ because $P(A|B) = \frac{P(A \cap B)}{P(B)}$, so $P(A|B) = \frac{3}{4}$.

2. B - here, $\lambda = \frac{1}{10}$ typos per page. Since we are interested in $p(1; \frac{1}{10})$, we solution is $e^{-\frac{1}{10}} \left(\frac{1}{10}\right)^1$.

3. B - for this problem we are interested in the probability of one or more arrivals, which is $1 - P(0 \text{ arrivals})$, which equals $1 - e^{-\frac{1}{10}} \left(\frac{1}{10}\right)^0$. 

2. 3 points for question 1 and 1 point for each part of question 2. For the questions in #2, if they have the concepts there but got a wrong number because, say, they looked at the wrong value in the chart, give them the one point anyway. If they just have a number, zero points for the subproblem. If they have computed the standard normal distribution equivalent incorrectly, they should loose a point. If the concepts are correct, but they made a silly mistake like computing $\mu$ or $\sigma$ incorrectly, please take off one point for all of problem 2 and give them 2 points (assuming they got correct answers and standard normal forms based upon their incorrect $\mu$ and $\sigma$ values).

1. The binomial distribution equation is given as:

$$b(k; n, p) = \binom{n}{k} \cdot (p)^k \cdot (1 - p)^{n-k}$$

Hence, the solution is:

$$\left(\frac{10,000}{6,500}\right) \cdot (0.6)^{6,500} \cdot (0.4)^{3,500}$$

For full credit the student must show the above equation. If they got close, but accidentally mixed up the values, like put 0.63500 and 0.46500, please award one point.

2. When using the normal distribution approximation, $\mu = np$ and $\sigma = \sqrt{npq}$ where $q = (1 - p)$. For this particular problem, then, $\mu = (10,000 \cdot 0.6) = 6,000$ and $\sigma = \sqrt{10,000 \cdot 0.6 \cdot 0.4} = 49$. For each of these problems, the distribution must be standardized in order to use the supplied table.

a. We want to find $P(6,000 \leq H \leq 6,100)$. Standardizing this we arrive at wanting to find the probability that $P\left(\frac{6,000-6,000}{49} \leq T \leq \frac{6,100-6,000}{49}\right) = P(0 \leq T \leq 2.04)$, which is 0.4793 by the table.

b. Here we want to find $P(H > 6,100)$, or, standardized, $P(T > 2.04)$. This is simple $0.5 - P(0 \leq T \leq 2.04)$, or 0.0207.

c. Here we want to find $P(5,975 \leq H \leq 6050)$, which, standardized, is $P(-0.51 \leq T \leq 1.02)$. This can be rewritten as $P(0 \leq T \leq 0.51) + P(0 \leq T \leq 1.02)$ which, by the table, is: 0.1950 + 0.3461 = 0.5411.
3. The students should have a diagram similar to the one provided. If they did not provide a diagram or any justification, but got the right answer, please award only 2 points. Note that they could compute this answer without a diagram by using Bayes’ Law. If they did this, and did it correctly, just take off one point, since the problem did ask for a diagram. The diagram in part 1 is sufficient justification for the solutions to parts 2 and 3. Please be kind when awarding partial credit. If the student is along the right path, but makes a silly math error, just subtract a point or two, as you see fit.

1. \( P(\text{x is blue}) = \frac{5}{30} + \frac{2}{30} + \frac{3}{30} = \frac{10}{30} = \frac{1}{3} \)

2. \( P(\text{Jar 2 was chosen from — x is red}) = \frac{P(\text{J2 chosen AND x is red})}{P(\text{x is red})} \). This is equal to \( \frac{8}{30} \cdot \frac{20}{30} = \frac{2}{5} \).

3. This is equal to:

\[
P(\text{y is red given that x is red}) = \frac{P(\text{x and y are red})}{P(\text{x is red})}
\]

which equals:

\[
\frac{20}{270} \cdot \frac{56}{270} + \frac{42}{270} = \frac{118}{270} = \frac{59}{90}
\]

If the student gets the correct answers and shows enough work to present a semi-logical trail you can follow, award full points. If NOT, assign partial credit as instructed.

4. (17 Points)
   a. Recursion (7 Points)
      \[ a_n = 2a_{n-1} + 2a_{n-2} \] (5 points)

      This formula is fairly simply derived: the number of ways to make a stack \( n \) inches high is to take all the stacks \( n - 1 \) inches high and add either a blue or red 1-inch block to the top, or take all the stacks \( n - 2 \) inches high and add either a blue or red 2-inch block. (2 points, must mention adding 2 different color blocks to each solution of size \( a_{n-1} \) and \( a_{n-2} \))

   b. Formula (10 Points)
      The solutions to the equation \( x^2 - 2x - 2 = 0 \) (3 points for characteristic equation) are the \( r_1 \) and \( r_2 \) of Theorem 4 from the book for deriving this formula, and are \( \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3} \). For our purpose, we’ll assign \( r_1 \) the plus and \( r_2 \) the minus (3 points for the roots). We then need to solve the system of equations \( A + B = 1 \) and \( Ar_1 + Br_2 = 2 \) to be able to get numbers for the final formula \( a_n = Ar_1^n + Br_2^n \).

      Solve the second equation for \( B \):
\[ A(1 + \sqrt{3}) + B(1 - \sqrt{3}) = 2 \]
\[ B(1 - \sqrt{3}) = 2 - A(1 + \sqrt{3}) \]
\[ B = \frac{2 - A(1 + \sqrt{3})}{1 - \sqrt{3}} \]

Replace B with the result and solve the first equation (1 point for solving correctly for A):

\[ A + \frac{2 - A(1 + \sqrt{3})}{1 - \sqrt{3}} = 1 \]
\[ A(1 - \sqrt{3}) + 2 - A(1 + \sqrt{3}) = 1 - \sqrt{3} \]
\[ A(1 - \sqrt{3}) - A(1 + \sqrt{3}) = -1 - \sqrt{3} \]
\[ A(1 - \sqrt{3}) + A(-1 - \sqrt{3}) = -1 - \sqrt{3} \]
\[ A(-2\sqrt{3}) = -1 - \sqrt{3} \]
\[ A = \frac{-1 - \sqrt{3}}{-2\sqrt{3}} = \frac{1 + \sqrt{3}}{2\sqrt{3}} \]

Now solve the first equation for B using the value of A (1 points for solving correctly for B):

\[ B = 1 - A = 1 - \frac{1 + \sqrt{3}}{2\sqrt{3}} \]
\[ \frac{2\sqrt{3}}{2\sqrt{3}} - \frac{1 + \sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3} - 1}{2\sqrt{3}} \]

So plugging in the appropriate values, the formula is (2 points for correct formula using whatever roots/coefficients you got previously):

\[ a_n = \left(\frac{1 + \sqrt{3}}{2\sqrt{3}}\right) \cdot (1 + \sqrt{3})^n + \left(\frac{\sqrt{3} - 1}{2\sqrt{3}}\right) \cdot (1 - \sqrt{3})^n \]

The two sum terms can be in any order, as long as each value to the \(n\) power is paired with the correct coefficient.

It’s possible to continue to “simplify” this formula to:

\[ a_n = \frac{(1 + \sqrt{3})^{n+1}}{2\sqrt{3}} - \frac{(1 - \sqrt{3})^{n+1}}{2\sqrt{3}} \]

This also gets full credit (2 points, in addition to what was gained above).
5. (23 Points)

a. (2 points)

\[ P(HH) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \]
\[ P(HT) = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9} \]
\[ P(TH) = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9} \]
\[ P(TT) = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} \]

b. (3 points)

\[ \text{image}(X) = \{0, 1, 2\} \]
\[ f_X(0) = P(TT) = \frac{4}{9} \]
\[ f_X(1) = P(HT) + P(TH) = \frac{2}{9} + \frac{2}{9} = \frac{4}{9} \]
\[ f_X(2) = P(HH) = \frac{1}{9} \]

Any clear matching set of pairs designating the distribution function \( f_X \) will work, as long as it’s correct.

c. (2 points)

\[ E(X) = (0 \cdot \frac{4}{9}) + (1 \cdot \frac{4}{9}) + (2 \cdot \frac{1}{9}) = \frac{2}{3} \]

The first term can be left out, since it’s obviously zero.

d. (3 points)

\[ Var(X) = E(X^2) - (E(X))^2 = \frac{8}{9} - \frac{4}{9} = \frac{4}{9} \]
\[ E(X^2) = (0 \cdot \frac{4}{9}) + (1 \cdot \frac{4}{9}) + (4 \cdot \frac{1}{9}) = \frac{8}{9} \]

If the variance was obtained using a different (but valid) formula, that’s fine too.
e. (4 points)
The easiest way I can think of to fill in the joint distribution is to set up the table and enter the values of $U$ wherever they fit in the table. $TT$ goes in the \{X = 0, Y = 0\} slot, $TH$ goes in the \{X = 1, Y = 0\} slot, $HT$ goes in the \{X = 1, Y = 1\} slot, and $HH$ goes in the \{X = 2, Y = 1\} slot. The other two are zero, since those combinations of the two variables are impossible.
Once you’ve filled that in, you’ll notice that the answer is (a).

f. (4 points)

$$Cov(X, Y) = E(XY) - E(X)E(Y) = \frac{4}{9} - \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$$

$$E(Y) = (0 \cdot \frac{2}{3}) + (1 \cdot \frac{1}{3}) = \frac{1}{3}$$

$$E(XY) = ((0 \cdot 0) \cdot \frac{4}{9}) + ((1 \cdot 0) \cdot \frac{2}{9}) + ((1 \cdot 1) \cdot \frac{2}{9}) + ((2 \cdot 1) \cdot \frac{1}{9}) = \frac{4}{9}$$

If the covariance is obtained using a different (but valid) formula, that’s fine too.

g. (5 points)

$$\rho(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}} = \frac{2/9}{2/3 \cdot \sqrt{2/3}} = \frac{1}{\sqrt{2}} = 0.707$$

$$Var(Y) = E(Y^2) - (E(Y))^2 = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$$

$$E(Y^2) = (0 \cdot \frac{2}{3}) + (1 \cdot \frac{1}{3}) = \frac{1}{3}$$

If the correlation is obtained using a different (but valid) formula, that’s fine too.