CSE 21 Homework 1

This is an individual homework assignment. Please answer all questions to the best of your ability, showing all work. Each problem should be done on a separate piece of paper, and each sheet of paper should have your name and student number on it. This homework is due on Tuesday the 8th.

Please realize that FOUR of these six problems are different from the ones posted on the Web site on Tuesday, October 1st. Please make sure you are doing the correct six problems (which are shown below) as opposed to the deleted ones from the previous assignment. If you have any questions please ask one of the TAs.

1. If we wish to partition a set $S$ into $k$ ordered partitions $S_1, S_2, \ldots, S_k$, where $m_i = |S_i|$ and $n = |S|$, then the number of such partitions is given as $\frac{n!}{m_1!m_2!\cdots m_k!}$. Prove that this equation holds using the Rule of Products. (Hint: Recall that the Rule of Products deals with solving a problem by breaking it down into $j$ steps and computing how many choices there are at each step. What step are we needing to perform, and how many of them? Partitions are defined in Example 23...)

2. If UCSD is offering 15 courses next quarter and you want to take 4, how many different course loads could you have? (Here order of the classes is not important, meaning taking courses A, B, C, and D is synonymous to taking courses C, D, B, and A.)

3. A phone number is made up of 10 digits, such as 858-123-4567. Answer the following questions, please:
   a. Are the 10 digits of a phone number a set, multiset, or list? Why?
   b. Assuming any digit (0-9) is valid in any position, how many unique phone numbers are possible?
   c. Assuming that the first digit of the 10 digit number cannot start with a 0, how many unique phone numbers are possible?
   d. Assuming that the first and fourth digits cannot start with 0, how many unique phone numbers are possible?
   e. Assuming that the first and fourth digits cannot start with 0, and the first three digits cannot be 911, how many unique phone numbers are possible?
   f. Assuming any digit (0-9) can appear in any position, how many phone numbers are there that have at least three 2’s?
4. Find the number of 14-character words formed by the numbers 1, 2, 3, 4, 5, 6, and the letters A, B, C, D, E, F, G, H without repetition of any letter or number. Find the number of words where the first three letters are A, B and C, in any order. Find the number of words where all letters are together (adjacent) and all numbers are together.

5. In how many ways can you invite five of your 8 friends to a party:
   a. If 2 of them are feuding and will not attend together.
   b. If 2 of them are married and will only attend together.

6. Prove that \( \sum_{i=0}^{n} \binom{n}{i} = 2^n \) and provide a combinatorial explanation. (Hint: Think of a set of \( n \) items and subsets of it.)