I. Overview of terms covered this week...

A. Given a function \( f : A \rightarrow B \), the image \( f \) is the subset of \( B \) that is mapped to by \( f \).

Question: If a function \( f \) is a surjection, what is the value of range\( (f) - \text{image}(f)\)?

Answer: the empty set

B. Earlier we saw that only bijections have inverse functions. However, we can define an inverse image to exist on any function, whether or not it is a bijection. Specifically, given a function \( f : A \rightarrow B \), the inverse image of \( f \) is defined as follows:

\[
 f^{-1}(b) = \{ a | a \in A \text{ and } f(a) = b \}.
\]

C. A random variable is a function that maps elements of the universal set \( U \) to real numbers. Formally, a random variable is a function \( X \), where \( X : U \rightarrow \mathbb{R} \).

D. A distribution function for a random variable \( X \) is defined as \( f_X : \text{image}(X) \rightarrow \mathbb{R} \), where for every \( t \in \text{image}(X) \), \( f_X(t) = P(X^{-1}(t)) \).

E. The expectation of a random variable \( X \) is defined as:

\[
 E(X) = \sum_{u \in U} X(s) \cdot P(s)
\]

this can also be written as:

\[
 E(X) = \sum_{r \in \text{image}(X)} r \cdot f_X(r)
\]

F. The variance of a random variable \( X \) is defined as:

\[
 Var(X) = \sum_{r \in \text{image}(X)} (r - E(X))^2 \cdot f_X(r)
\]

Note that the expectation doesn’t tell us anything about how the data points vary. That is, if someone says that the average income in California is 50,000, you don’t know if most people make around 50,000 or if half of the people make 100,000 and half make 0. The variance gives you an idea of how the data is spread apart.

G. The standard deviation is simply the square root of the variance.

II. Examples!

A. The State of California has a lottery game where the player picks five numbers out of a field of 39 numbers (1 through 39). To play this game, players must pay one dollar. The winning number is drawn from 39 numbers without replacement. Assume that if
you get all 5 numbers correct, you win $50,000. If you get 4 numbers correct, you get $5,000. If you get 3 numbers correct, you get $1,000. If you get 2 numbers correct, you get your dollar back. (Hence, if you get 0 or 1 numbers correct, you lose a dollar.)

Now, let’s answer some questions:

1. What is the universal set $U$?
2. What is the probability of getting all five numbers correct? What about 4? 3? 2?
3. Let $X$ be a random variable that maps possible number picks to their winning outcome. What is the $\text{image}(X)$?
4. What is the distribution function of $X$?
5. What is the expectation of $X$?
6. Given the expectation value, who benefits from the lottery? Those who play it, or the State of California?