Practice problem on semantic optimization

Let $R$ be a relation over attributes $ABC$.

(i) Simplify the following query, knowing that it is applied only to relations $R$ satisfying the fds $\{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$ (use tableau minimization and the chase):

$$q = \pi_{AB}[\pi_{AC}(R) \bowtie \pi_{CB}(R)] \bowtie \pi_{BC}[\pi_{BA}(R) \bowtie \pi_{AC}(R)].$$

(ii) Redo (i) for the query:

$$q = \pi_{AB}[\pi_{AC}(R) \bowtie \pi_{CB}(\sigma_{C=5}(R))] \bowtie \pi_{BC}[\pi_{BA}(R) \bowtie \pi_{AC}(\sigma_{A=8}(R))].$$

Solution  

(i) The tableau $T$ corresponding to $q$ is

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
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</thead>
<tbody>
<tr>
<td>a</td>
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<td>b</td>
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<td>c</td>
</tr>
</tbody>
</table>

Chasing with respect to $\Sigma = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$ yields:

$$CHASE_{\Sigma}(T) = \begin{array}{ccc}
A & B & C \\
a & b & c \\
a & b & c \\
a & b & c \\
a & b & c \\
a & b & c \\
a & b & c \\
a & b & c \\
\end{array}$$

Eliminating duplicate rows from $CHASE_{\Sigma}(T)$ yields the minimal tableau:

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<thead>
<tr>
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<th>B</th>
<th>C</th>
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</thead>
<tbody>
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</tbody>
</table>

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and the corresponding rspj query is
\[ \pi_{AB}(R) \Join \pi_{AC}(R). \]

(ii) The tableau \( T \) corresponding to \( q \) is

\[
\begin{array}{ccc}
A & B & C \\
\hline
a & b_1 & 5 \\
a_1 & b & 5 \\
8 & b & c_2 \\
8 & b_2 & c \\
a & b & c
\end{array}
\]

Chasing with respect to \( \Sigma = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\} \) yields (after eliminating duplicate rows):

\[
CHASE_\Sigma(T) = \begin{array}{ccc}
A & B & C \\
\hline
8 & b & 5 \\
8 & b_2 & c \\
8 & b & c
\end{array}
\]

This tableau is minimal, and the corresponding rspj query is
\[ \pi_{AB}(\sigma_{A=8}(\sigma_{C=5}(R))) \Join \pi_{AC}(\sigma_{A=8}(R)). \]