Midterm 2, CSE21, Fall 2001

Student Name: ____

Instructions: Begin by writing your name above.

This is a closed book and notes exam. The exam includes the table for calculating probabilities using the normal distribution. You may not use calculators.

Try all problems. Write your answers on the exam sheet itself, in the assigned space below the problem, or on the extra blank page that follows.

Suggestions: Scan through the entire exam before beginning any question. You don't have to do the questions in order: do first whatever you find easiest.

Guidelines: Remember to write clearly: answers that don't make sense will not get partial credit. You are graded on what you write, not on what you think you mean! So read carefully what you write.

1. indicate with a brief note the application of the Rule of Product, Rule of Sum, inclusion-exclusion principle.

2. When a random variable of interest follows the binomial, or Poisson or normal distribution, write the distribution of the random variable. For example, write "variable X follows the normal distribution with mean 100 and standard deviation 10" or "variable X follows the binomial distribution $b(k; n, p)$". Such notes will help you get partial credit if the final answer is wrong. When computing probabilities in equiprobable spaces indicate the size of the universal set and the size of the event set.

Important Note: For all problems the following types of answers are OK:

- Formulas involving $n!$, $\binom{n}{k}$, $n \cdot m \cdot k$,
- the Poisson distribution $p(k; \lambda) = e^{-\lambda} \frac{\lambda^k}{k!}$

HOWEVER, in multiple choices that ask for the specific number, calculations have to be done.

Good luck!

Scores: To be filled in at grading time.

On Problem	You got	Out of
1		25
2		20
3		20
Extra Credit		15
Total:		80

1. Multiple choice questions. Every correct answer gains 4 or 5 points. Every wrong answer costs 1 point.

   (a) **Win 4, lose 1** Consider a random variable X such that $image(X) = \{-1, 0, 1\}$ and $E(X) = 2$. Then $E(2X + 1) =$

      i. 2
      ii. 5
      iii. 1
      iv. cannot be determined from the given information
      v. none of the above

   (b) **Win 4, lose 1** A random variable X follows the normal distribution with mean 10 and standard deviation 2. Then $P(8 \leq X \leq 12)$ is

      i. 0.4000
      ii. 0.3413
      iii. 0.4772
      iv. 0.6826
      v. 0.9544

   (c) **Win 4, lose 1** Let us consider again the random variable X that follows the normal distribution with mean 10 and standard deviation 2. Then $P(8 \leq X)$ is

      i. 0.8413
      ii. 0.6000
      iii. 0.9544
      iv. 0.3413
      v. 1.3413

   (d) **Win 4, lose 1** Carla on the average gets a flu once every 12 months. What is the probability that she will get the flu in November?

      i. $\frac{11}{12}$
      ii. $e^{-\frac{1}{12}}\frac{11}{12}$
      iii. $e^{-\frac{1}{12}}\frac{1}{12}$
      iv. $1 - e^{-\frac{1}{12}}$
      v. $e^{-\frac{1}{12}}$

   (e) **Win 4, lose 1** Carla on the average gets a flu once every 12 months. What is the probability that she will not get the flu in November? (Hint: $0! = 1$)

      i. $\frac{11}{12}$
      ii. $e^{-\frac{1}{12}}\frac{11}{12}$
      iii. $1 - e^{-\frac{1}{12}}\frac{1}{12}$
      iv. $1 - e^{-\frac{1}{12}}$
      v. $e^{-\frac{1}{12}}$

(f) **Win 5, lose 1** We toss a fair die. The outcome of the experiment is a number from $U = \{1, 2, 3, 4, 5, 6\}$ and all outcomes are equally probable. Next we define the random variables $X : U \mapsto R$ and $Y : U \mapsto R$ as follows:

$$X(t) = \{ \begin{array}{l} 0, \ if \ t \in \{1, 2, 3\} \\ 1, \ if \ t \in \{4, 5, 6\} \end{array}$$

and

$$Y(t) = \{ \begin{array}{l} 0, \ if \ t \in \{1, 3, 5\} \\ 1, \ if \ t \in \{2, 4, 6\} \end{array}$$

The joint distribution $h_{X,Y}$ of X and Y is

i.

$h_{X,Y}$	$X = 0$	$X = 1$
$Y = 0$	$\frac{1}{2}$	$\frac{1}{2}$
$Y = 1$	$\frac{1}{2}$	$\frac{1}{2}$

ii.

$h_{X,Y}$	$X = 0$	$X = 1$
$Y = 0$	$\frac{2}{6}$	$\frac{1}{6}$
$Y = 1$	$\frac{1}{6}$	$\frac{2}{6}$

iii.

$h_{X,Y}$	$X = 1$	$X = 2$	$X = 3$	$X = 4$	$X = 5$	$X = 6$
$Y = 1$	$\frac{1}{6}$	0	0	0	0	0
$Y = 2$	0	$\frac{1}{6}$	0	0	0	0
$Y = 3$	0	0	$\frac{1}{6}$	0	0	0
$Y = 4$	0	0	0	$\frac{1}{6}$	0	0
$Y = 5$	0	0	0	0	$\frac{1}{6}$	0
$Y = 6$	0	0	0	0	0	$\frac{1}{6}$

iv. none of the above

v. cannot be determined from the given information

2. John plays card games at a Las Vegas casino room for novices.

   (a) **10 points** John draws a card from a standard deck of 52 cards. Recall, a deck of 52 cards has exactly 13 spades and exactly four aces, one of which is the "Ace of Spades".

      i. if the card is an ace John wins \$13, even if it is the "Ace of Spades"

      ii. if the card is a spade, other than the "Ace of Spades", then John loses \$5.

      Define a random variable X that corresponds to the dollars won or lost in each round and then find how much money John wins or loses on the average in each round.

   (b) **10 points** Now John plays a riskier game: A card is drawn and replaced 10 times from an ordinary deck of cards ("replace" means that the card is put back into the deck.)

      i. If John draws an ace exactly 3 times then he wins \$25

      ii. otherwise he loses \$1

      How much money does John win or lose on the average in each round?

3. **20 points** Consider an experiment where a pair of fair dice is thrown. The universal set U is the set of all pairs of outcomes of the two dice, i.e.,

$$U = \{[x, y] \mid x, y \in \{1, 2, 3, 4, 5, 6\}\}$$

Let $X : U \mapsto R$ be the random variable that denotes the absolute value of the difference of the two dice, that is

$$X([x, y]) = |x - y|$$

For example,

(a) if the first die turns out to be 5 and the second turns out to be 1, then

$$X([5, 1]) = |5 - 1| = 4$$

(b) if the first die turns out to be 1 and the second turns out to be 5, then

$$X([1, 5]) = |1 - 5| = 4$$

(c) if the first die turns out to be 5 and the second turns out to be 5, then

$$X([5, 5]) = |5 - 5| = 0$$

Compute the distribution f_X, the expectation $E(X)$, the variance $Var(X)$, and the standard deviation σ_X.

4. **Extra 15 points** Suppose X is a random variable with distribution f_X and $image(X) = \{-1, 0, 1\}$, i.e., X may take the values $-1, 0, 1$. We know that $f_X(0) = \frac{1}{2}$ but we do not know $f_X(-1)$ or $f_X(1)$. We also know that $E(X) = \frac{1}{6}$. Compute $Var(X)$.

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(Don't forget to indicate the problems you solve here)