CSE21 HW #7 Solution

1. (15 Points) Let X be a binomially distributed random variable, with mean 12 and variance 4.8. Find $P(X > 5)$ and $P(5 < X < 10)$

   Solution

   A binomially distributed random variable has a mean np and variance npq, where $q = 1 - p$. Then we have the following system of two equalities and two unknowns:

   $np = 12$
   $np(1 - p) = 4.8$

   We solve for n and p and get $p = 0.6$ and $n = 20$.

   $P(X > 5) = 1 - P(X \leq 5)$
   $P(X \leq 5) = \sum_{i=0}^{5} P(X = i) = \sum_{i=0}^{5} C(20, i) \times (p^i q^{n-i}) = \sum_{i=0}^{5} \frac{20!}{i!(20-i)!} \times (0.6^i 0.4^{20-i}) = 0.002$
   $P(X > 5) = 1 - P(X \leq 5) = 0.998$

   $P(5 < X < 10) = \sum_{i=6}^{9} P(X = i) = \sum_{i=6}^{9} C(20, i) \times (p^i q^{n-i}) = \sum_{i=6}^{9} \frac{20!}{i!(20-i)!} \times (0.6^i 0.4^{20-i}) = 0.126$

2. (15 Points) Let X be the binomial random variable $B(2, p)$. Let Y be the binomial random variable $B(4, p)$.
   $P(X \geq 1) = \frac{8}{9}$. Find $P(Y \geq 1)$.

   Solution

   We have $P(X \geq 1) = P(X = 1) + P(X = 2) = C(2, 1)p(1 - p) + C(2, 2)p^2$ and $P(X \geq 1) = \frac{8}{9}$. We solve for p and get $p = \frac{2}{3}$ or $p = \frac{4}{3}$. Because $p \leq 1$, we get $p = \frac{2}{3}$.

   Therefore Y is the binomial random variable $B(4, \frac{2}{3})$.
   $P(Y \geq 1) = 1 - P(Y < 1) = 1 - P(Y = 0) = 1 - C(4, 0)p^0(1 - p)^4 = 1 - (\frac{1}{3})^4 = \frac{80}{81}$.

3. (10 Points) Let Z be the random variable with the normal distribution with mean 0 and standard deviation σ^2.
   k satisfies $P(|Z| \geq k) = 0.1$. Find $P(Z < k)$.

   Solution

   We have $P(|Z| \geq k) = 0.1$ and $P(|Z| \geq k) = P(Z \geq k) + P(Z \leq -k) = 2P(Z \geq k)$.

   Therefore $P(Z \geq k) = 0.05$.

   $P(Z < k) = 1 - P(Z \geq k) = 1 - 0.05 = 0.95$.

4. (20 Points) Suppose the grades of an examination is normally distributed with mean $\mu = 70$ and standard deviation $\sigma = 16$. The top 15 percent of the students receive A's and the bottom 10 percent receive F's. Find:

   (a) the minimum grade to receive an A.

   (b) the minimum grade to pass (not to receive F).

   Solution

   (a) (10 pts)

Let X be normally distributed with mean $\mu = 70$ and standard deviation $\sigma = 16$. Let Z be the standard normal random variable. Let k be the minimum grade to receive an A, then $P(X \geq k) = 0.15$.

$$P(X \geq k) = P(Z \geq \frac{k-70}{16}) = 0.15 \Longrightarrow P(0 \leq Z < \frac{k-70}{16}) = 0.5 - 0.15 = 0.35$$

Look up Table 6-1 in the textbook, we have $\frac{k-70}{16} = 1.04$.
Therefore $k = 86.64 \approx 87$.

(b) (10 pts)
Let t be the minimum grade to pass, then $P(X \geq t) = 1 - 0.1 = 0.9$.
$P(X \geq t) = P(Z \geq \frac{t-70}{16}) = 0.9$
$\Longrightarrow P(\frac{t-70}{16} \leq Z \leq 0) = 0.9 - 0.5 = 0.4$
$\Longrightarrow P(0 \leq Z \leq -\frac{t-70}{16}) = 0.9 - 0.5 = 0.4$
Look up Table 6-1 in the textbook, we have $-\frac{t-70}{16} = 1.28$.
Therefore $t = 49.52 \approx 50$.