CSE21 HW #4 Solution

1. (10 Points) Two new drugs are to be tested using a group of 40 laboratory mice, each tagged with a number for identification purposes. Drug A is to be given to 15 mice, drug B is to be given to another 15 mice, and the remaining 10 mice are to be used as controls. How many ways can the assignment of treatments to mice be made? (A single assignment involves specifying the treatment for each mouse - whether drug A, drug B, or no drug.)

   Solution The assignment of treatments to mice can be considered a two-step operation. Step 1 is to choose 15 mice out of the 40 to receive treatment A, step 2 is to choose 15 mice out of the remaining 25 to receive treatment B. The remaining 10 mice are the controls.
   $C(40,15) \cdot C(25,15) = \frac{40!}{15!25!} \cdot \frac{25!}{15!10!} \approx 1.315 \times 10^{17}$.

2. (10 Points) Suppose that three computer boards in a production run of forty are defective. A sample of four is to be selected to be checked for defects.

   (a) How many different samples can be chosen?

   (b) How many samples will contain at least one defective board?

   (c) What is the probability that a randomly chosen sample of five contains at least one defective board?

   Solution

   (a) (3 pts) C(40, 4) = 91390

   (b) (3 pts) C(40,4) - C(37,4) = 91390-66045 = 25345

   (c) (4 pts) (C(40,5) - C(37,5))/C(40,5) = (658008-435897)/658008 = 0.338.
   For those who solved *what is the probability that a randomly chosen sample of four contains at least one defective board*, the solution should be (C(40,4) - C(37,4))/C(40,4) = 25345/91390 = 0.277. This will also get full points.

3. (10 Points) Your drawer contain 10 black, 20 red, and 15 blue socks. What is the probability that you take out (with your eyes closed)

   (a) a non-matching pair?

   (b) a pair of red socks?

   (c) a pair of blue sock?

   When answering, define the universal set and appropriate events.

   Solution Let the set of outcomes U be the set of all possible ways to take two socks out of the drawer. Then $U = P(45,2) = 45 \times 44$. We solve (b) and (c) first and then (a).
   (b) (3 pts) Let the E_b be the event that the two socks taken out are red. Then $|E_b| = P(20,2)$, and $P(E_b) = \frac{|E_b|}{|U|} = \frac{20 \times 19}{45 \times 44}$.
   (c) (3 pts) Let the E_c be the event that the two socks taken out are blue. Then $|E_c| = P(15,2)$, and $P(E_c) = \frac{|E_c|}{|U|} = \frac{15 \times 14}{45 \times 44}$.
   (a) (4 pts) Let the E_a be the event that the two socks taken out are black. Then $|E_a| = P(10,2)$, and $P(E_a) = \frac{|E_a|}{|U|} = \frac{10 \times 9}{45 \times 44}$.
   And let E be the event that the two sock taken out are different colors then
   $P(E) = 1 - (P(E_a) + P(E_b) + P(E_c))$.

4. (10 Points) There are 4 white, 3 red, and 3 blue balls in a bag. If you draw 3 balls, what's the probability that less than 2 will be blue if

   (a) You draw with replacement?

   (b) You draw without replacement?

   Solution

   (a) (5 pts) $(7^3 + 3*7*7 + 7*3*7 + 7*7*3)/10^3 = 98/125 = 0.784$

   (b) (5 pts) $(C(7,3) + C(3,1)C(7,2))/C(10,3) = 49/60 = 0.817$

5. (10 Points)

   (a) if $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.6$, $P(A \cap B) =$?

   (b) if $P(A) = P(B) = P(C) = 0.25$, $P(A \cap B) = P(B \cap C) = 0$ and $P(A \cap C) = 0.125$, $P(A \cup B \cup C) =$?

   Solution

   (a) (5 pts) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.4 + 0.3 - 0.6 = 0.1$

   (b) (5 pts) $0 \leq P(A \cap B \cap C) \leq P(A \cap B) = 0 \Longrightarrow P(A \cap B \cap C) = 0$

   $$\begin{aligned}
   &P(A \cup B \cup C) \\
   &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\
   &= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C)) \\
   &= P(A) + P(B) - P(A \cap B) + P(C) - (P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))) \\
   &= P(A) + P(B) - P(A \cap B) + P(C) - (P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \\
   &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \\
   &= 0.25 + 0.25 + 0.25 - 0 - 0 - 0.125 + 0 = 0.625
   \end{aligned}$$

6. (10 Points) There are six students in a group. What's the probability that

   (a) at least one of them has his/her birthday in October?

   (b) there are exactly four of them whose birthdays are in October?

   (c) there are exactly four of them whose birthdays are in the same month?

   Solution

   (a) (3 pts) $1 - (\frac{11}{12})^6 \approx 0.4067$.

   (b) (4 pts) $\frac{C(6,4) \cdot 11^2}{12^6} \approx 0.0006$.

   (c) (3 pts) $\frac{C(12,1) \cdot C(6,4) \cdot 11^2}{12^6} \approx 0.0073$.