# CSE21 Homework 1

Due: January 18, 2005

This is an **individual** homework assignment. Please answer all questions to the best of your ability, showing all work.

**For uniform grading, each question should be graded by only one reader. So please make sure that each problem is on a separate piece of paper, and that each piece of paper has your name and student ID number on it.**

1. $F_n$ $(n = 0, 1, 2, \ldots)$ are the Fibonacci numbers. Prove by mathematical induction: $F_n < \left(\frac{7}{4}\right)^n$ for all $n \geq 1$.

2. Find an explicit formula for the sequence $a_0$, $a_1$, $a_2$, $\ldots$, that satisfies

$$a_n = 4a_{n-1} + 8a_{n-2}, \text{ for all integers } n \geq 2$$

   with initial conditions $a_0 = 2$ and $a_1 = 2$.

3. Consider two dimensional space. If we draw a line it will divide it into two regions. Two lines would split the space into four regions. Three lines can divide it (if the third is not parallel to either of the previous two lines) into seven regions. Let $Pn$ be the maximum number of regions into which $n$ lines could divide a plane, where $n$ is a positive integer.

   (a) Derive a recurrence relation for $P_k$ in terms of $P_{k-1}$ for all integers $k \geq 2$.

   (b) Use the iteration method to guess and prove an explicit formula for $P_n$.

4. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

   Compute $A^n$ for $n = 2, 3, 4, 5$. Give a general (explicit) formula for the entries in the matrix and prove your answer correct.

5. A circular disk is cut into $n$ distinct sectors, each shaped like a piece of pie and all meeting at the center point of the disk. Each sector is to be painted red, green, yellow or blue in such a way that no two adjacent sectors are painted the same color. Let $S_n$ be the number of ways to paint the disk.

   (a) Find a recurrence relation for $S_k$ in terms of $S_{k-1}$ and $S_{k-2}$ for each integer $k \geq 4$.

   (b) Find an explicit formula for $S_n$ for $n \geq 2$.

6. Let $a_n$ be the number of the n-bit binary numbers that satisfy the following condition: None of the binary numbers contains 11 in its binary representation. E.g. The 4-bit binary number 0101 satisfies the condition, while 0110 does not.

   Find a recurrence relation for $a_n$ and solve it.