CSE 21 Mathematics for Algorithms and Systems
Discussion 4
Sample space. Events. Axioms and theorems of probability. Exercises.

February 7

Ex. 1 Soccer team has 20 German and 20 English players. The players are to be paired in groups of 2 (to share rooms). If the pairing is at random (equiprobable)

(a) What is the probability that there will be no $(Germain, English)$ pairs.

(b) What is the probability that there are $2i$ $(Germain, English)$ pairs, for $i = 0, 1, ..., 10$?

Solution

(a) We define the universal set U to be the set of all possible unordered pairs. Note that, if G_1, E_1 are two German and English players, respectively, then (G_1, E_1) is the same as (E_1, G_1). Number of lists of unordered pairs is $\frac{(20+20)!}{(2!...2!)} = \frac{40!}{2^{20}}$. And the size of the set of unordered pairs is $\frac{40!}{2^{20}(20!)}$.

$$|U| = \frac{40!}{2^{20}(20!)}$$

E is the event that the German players are paired between themselves and the $English$ players are paired between themselves. Then $P(E)$ will give us the desired probability, because no pair of the kind (G, E) will be present.

$|E| = ($ # ways to pair Germans among themselves$)\times($# ways to pair Englishmen among themselves$)$

We use the same counting technique for the number of ways to pair Germans (the case for the Englishmen is the same) as calculating U, exept the number of player is 20, the number of pairs is 10, therefore.

$$\text{\# ways to pair Germans among themselves} = \frac{20!}{(2!)^{10}(10!)}$$

$$|E| = (\frac{20!}{2^{10}(10!)})^2$$

The desired probability is $P(E) = \frac{|E|}{|U|}$

(b) The set of outcomes U remains the same. Note that, we have calculated E_0 previously. Let E_{2i} be the event that there are exactly $2i$ $(Germain, English)$ pairs.
$|E_{2i}| =$ (# of ways to pair $2i$ Germans to $2i$ Englishmen) $\times$
(# of ways to pair $20 - 2i$ Germans among themselves) $\times$
(# of ways to pair $20 - 2i$ Englishmen among themselves)

$$\text{\# of ways to pair } 2i \text{ Germans to } 2i \text{ Englishmen} = (2i)!$$

Counting the number of ways to pair Englishmen among themselves is the same as before, except the number of players is $20 - 2i$, the number of pairs is $10 - i$, thus we have:

$$|E_{2i}| = (2i)! \times (\frac{(20 - 2i)!}{((2!)^{10-i}(10 - i)!})^2$$

$$P(E_{2i}) = \frac{|E_{2i}|}{|U|}$$