Midterm 1, CSE21, Fall 2001

Student Name: ____

Instructions: Begin by writing your name above.

This is a closed book and notes exam. You may not use calculators.

The exam contains one problem with seven questions, one problem with nine questions and an extra credit problem. Try them all. Write your answers on the exam sheet itself, in the assigned space below the problem, or on the extra blank page that follows.

Suggestions: Scan through the entire exam before beginning any question. You don't have to do the questions in order: do first whatever you find easiest. We suggest you do the extra credit problem last, due to its difficulty.

Guidelines: Remember to write clearly: answers that don't make sense will not get much credit. You are graded on what you write, not on what you think you mean! So read carefully what you write. Also, indicate with a brief note the application of the Rule of Product, Rule of Sum, and inclusion-exclusion principle. When computing probabilities in equiprobable spaces indicate the size of the universal set and the size of the event set.

Important Note: For all problems answers like $n!$, $\binom{n}{k}$, $n \cdot m \cdot k$ are fine. HOWEVER, in multiple choices that ask for the specific number, calculations have to be done.

Good luck!

Scores: To be filled in at grading time.

On Problem	You got	Out of
1		28
2		43
3		9
Total:		80

1. **Win 24 points in multiple choice!** Every correct answer gains 4 points. Every wrong answer costs 1 point.

   (a) **Win 4, don't lose 1** How many 4-letter words can be formed from the letters of "answer", if each letter appears at most once?

      i. $6 \cdot 5 \cdot 4 \cdot 3$
      ii. $\frac{6 \cdot 5 \cdot 4 \cdot 3}{2 \cdot 1}$
      iii. $\frac{6 \cdot 5 \cdot 4 \cdot 3}{(4 \cdot 3 \cdot 2 \cdot 1)(2 \cdot 1)}$
      iv. cannot tell given the above information
      v. none of the above

   (b) **Win 4, don't lose 1** Let $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{1}{2}$. Then $P(A \cap B) =$

      i. $\frac{2}{3}$
      ii. $\frac{1}{6}$
      iii. $\frac{1}{3}$
      iv. cannot tell given the above information
      v. none of the above

   (c) **Win 4, don't lose 1** The number of ways to distribute 5 toys to 3 children so that the first child gets 3 toys, the second gets one and the third gets one, is:

      i. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
      ii. $5 \cdot 4 \cdot 3 + 2 + 1$
      iii. $\frac{5 \cdot 4 \cdot 2 \cdot 1}{2}$
      iv. cannot tell given the above information
      v. none of the above

   (d) **Win 4, don't lose 1** A student group contains 4 girls and 3 boys. Yannis randomly chooses a committee of 4 students. What is the probability that the committee contains exactly 2 girls and 2 boys?

      i. $\frac{18}{35}$
      ii. $\frac{18}{210}$
      iii. $\frac{90}{210}$
      iv. cannot tell given the above information
      v. none of the above

(e) **Win 4, don't lose 1** Consider the function f from the domain set $\{a, b, c\}$ to the range set $\{1, 2, 3\}$, such that $f(a) = 1, f(b) = 3, f(c) = 3$. Is the function f a

  i. surjection
  ii. injection
  iii. bijection
  iv. cannot tell given the above information
  v. none of the above

(f) **Win 4, don't lose 1** Consider again the function f from the domain set $A = \{a, b, c\}$ to the range set $B = \{1, 2, 3\}$, such that $f(a) = 1, f(b) = 3, f(c) = 3$. The generalized inverse $f^{-1} : B \mapsto \mathcal{P}(A)$, in the two-line notation is

  i. $\left(\begin{array}{ccc} 1 & 2 & 3 \\ \{a\} & \{\} & \{b, c\} \end{array} \right)$
  ii. $\left(\begin{array}{ccc} a & b & c \\ \{a\} & \{\} & \{b, c\} \end{array} \right)$
  iii. $\left(\begin{array}{cccc} 1 & 2 & 3 & 3 \\ a & \{\} & b & c \end{array} \right)$
  iv. it is impossible to define
  v. none of the above

(g) **Win 4, don't lose 1** Consider two functions f and g, where f is defined as: $f(1) = 5$,$f(2) = 4$,$f(3) = 1$,$f(4) = 3$,$f(5) = 2$, and g is defined as: $g(1) = 2$,$g(2) = 3$,$g(3) = 1$,$g(4) = 4$,$g(5) = 5$. The $(f \circ g)$, or (fg), is:

  i. $(fg)(1) = 1$,$(fg)(2) = 2$,$(fg)(3) = 3$,$(fg)(4) = 4$,$(fg)(5) = 5$
  ii. $(fg)(1) = 5$,$(fg)(2) = 4$,$(fg)(3) = 3$,$(fg)(4) = 2$,$(fg)(5) = 1$
  iii. $(fg)(1) = 4$,$(fg)(2) = 1$,$(fg)(3) = 5$,$(fg)(4) = 3$,$(fg)(5) = 2$
  iv. $(fg)(1) = 2$,$(fg)(2) = 3$,$(fg)(3) = 5$,$(fg)(4) = 1$,$(fg)(5) = 4$
  v. none of the above

2. **Win 43 points in card games!** For this problem you do not have to compute the final number answers.

   Consider a hand of 5 cards drawn from a deck of 52 cards.

   (a) **6 points** What is the probability that the hand contains exactly 2 aces?

   (b) **5 points** What is the probability that the hand contains at least 2 aces?

   (c) **3 points** What is the probability that the hand contains exactly 2 aces, the "Queen of Diamonds" and two other cards that are not aces?

(d) **3 points** What is the probability that the hand contains exactly 2 aces, the "Queen of Diamonds" or the "Queen of Hearts" (but not both) and two other cards that are not aces?

(e) **5 points** What is the probability that the hand contains 2 aces, 2 kings, and 1 queen?

(f) **3 points** What is the probability that the hand contains exactly 2 aces, exactly 2 kings, and a fifth card that is not an ace or a king?

(g) **6 points** What is the probability that the hand contains exactly 2 aces or exactly 2 kings? (Hint: It's not exclusive or)

(h) **6 points** What is the probability that the hand contains at least 2 aces and at least 2 kings? Notice that the fifth card may or may not be an ace or a king.

(i) 6 points What is the probability that the hand contains 5 different values? A set of 5 different values could be {"5 of Diamonds", "6 of Clubs", "2 of Diamonds", "Ace of Spades", "Queen of Clubs"}.

3. **Extra Credit Problem (9 points)** There are 6 different colored pairs of socks [1 green, 1 blue, 1 red, 1 yellow, 1 orange, 1 purple], 12 socks in all.

   - **(a) 4 points** If 2 socks are selected at random, what is the probability that the socks are a matching pair, that both are the same color?

   - **(b) 5 points** If the 12 socks are to be distributed into 6 boxes, each of which is to contain exactly 2 socks, what is the probability that all boxes contain matched pairs (ie. the red pair is in one box, the blue pair is in another, etc.)? (Hint: It might help to think that the boxes are ordered)

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(Don't forget to indicate the problems you solve here)