Practice Problems for the Second Midterm of CSE21, Part B

November 5, 2001

Some of the following problems have indeed been given in past midterms and final exams of CSE21. Others are problems that we wrote for the midterm review session.

1. Multiple choice problems

(a) Suppose \( X \) is a random variable that takes the values 0 and 1 and \( E(X) = \frac{1}{2} \).
   i. \( P(X = \frac{1}{2}) = 1 \)
   ii. \( P(X = 1) = \frac{1}{2} \)
   iii. none of the above

(b) We toss a fair coin 100 times. The probability to get heads 50 times is
   i. \( 50/100 \)
ii. \( \binom{100}{50} \left( \frac{1}{2} \right)^{50} \)

iii. \( \binom{100}{50} \left( \frac{1}{2} \right)^{100} \)

iv. none of the above

(c) \( X \) is a random variable with \( E(X) = 2 \) and \( Var(X) = \frac{1}{3} \).

i. \( P(X > 2) = \frac{1}{2} \)

ii. \( P(X > 2) = \frac{1}{3} \)

iii. cannot be determined from the above

2. A player tosses three fair coins. He wins $15 if 3 heads occur, $6 if two heads occur, and $2 if 1 head occurs. If the game is to be fair, how much should he lose if no heads occur?
3. Let $X$ be a continuous random variable with probability density function

$$f_x(x) = \begin{cases} \frac{k}{x}, & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $k$ is a constant. Determine the value of $k$.

4. A fair die is tossed. Let $X$ denote twice the number appearing, and let $Y$ denote 1 or 3 according as an odd or an even number appears. Find the distribution, expectation, variance and standard deviation of

(a) $X$,
(b) $Y$,
(c) $X + Y$.

5. A player tosses two fair coins. He wins $1$ or $2$ according as 1 or 2 heads appear. On the other hand, he loses $5$ if no heads appear.
Design a suitable random variable $X$ and determine the expected value $E(X)$ of the game and if it is favorable to the player.

6. Let $X$ be a random variable with mean $\mu$ and standard deviation $\sigma > 0$; and let $X^*$ be the standardized random variable corresponding to $X$, i.e. $X^* = (X - \mu)/\sigma$. Show that $E(X^*) = 0$ and $Var(X^*) = 1$. (Hence $\sigma_{X^*} = 1$.)