Practice Problems for the Second Midterm of CSE21, Part A

November 4, 2001

Some of the following problems have indeed been given in past midterms and final exams of CSE21. Others are problems that we wrote for the midterm review session.

1. Multiple choice problems:

   (a) Suppose $X$ is a random variable with distribution $f_X$, such that $f_X(2) = P(X^{-1}(2)) = 0.3$, and $f_X(4) = P(X^{-1}(4)) = 0.7$. Then $E(X) =$

   i. 2.7
   ii. 3
   iii. 3.4
   iv. cannot be determined from the given information

   (b) Suppose $X$ is a random variable which equals 2 with probability 0.8, and equals 1 with probability 0.2. Then $Var(X)$
i. is $1.8^2$
ii. is 0.16
iii. is 3.4
iv. cannot be determined from the given information

(c) Suppose $X$ are random variables such that $E(X) = 0.5$ and $E(Y) = 0.2$. Then $E(XY)$
   i. is 0.1
   ii. is 0.6
   iii. is 0.7
   iv. cannot be determined from the given information

(d) Suppose $X, Y$ are random variables such that $E(X) = 0.5$ and $E(Y) = 0.2$. Then $E(X + Y)$
   i. is 0.7
   ii. is 0.6
   iii. is 0.1
iv. cannot be determined from the given information

(e) Suppose $X$ is a random variable which only takes values 0 or 1, and $E(X) = 0.3$. Then $Var(X)$

i. is 0.3

ii. is 0.21

iii. cannot be determined from the given information

(f) Suppose we flip, one million times, a coin which has probability $1/4$ of being heads (H) and $3/4$ of being tails (T). Then the expected number of heads is

i. close to zero

ii. close to 250,000

iii. $\left(\frac{1000000}{250000}\right) \cdot \left(\frac{1}{4}\right)^{250000} \cdot \left(\frac{3}{4}\right)^{750000}$
2. Alice and Bob each toss a fair die, and Alice wins if the value on her die is greater than or equal to the value on Bob’s die.

(a) Whenever Alice wins she receives $1. Whenever she loses she pays $2. Should Alice be playing this game with Bob?

(b) If they play 35 times, what is the probability that Alice wins exactly 20 games?

(c) If they play 50 times, what is the probability that Alice wins at least 30 games?
3. Consider a continuous random variable \( X \) following the normal distribution \( N(100, 10^2) \). Compute the probability

\[
P(90 \leq X \leq 120)
\]

You may use the table that was handed out. The table provides the probability \( P(0 \geq X^* \geq t) \) where the variable \( X^* = \frac{X-100}{10} \) follows the standard normal distribution \( N(0, 1) \).

4. Alice is going to Las Vegas for the weekend to play a game which has the following features. Every time Alice plays, she has

- A 10\% chance of winning $12,
- A 50\% chance of winning nothing
- A 40\% chance of losing $3.

Let \( X \) be the random variable that represents the amount of money Alice will win or lose at the end of a 100-game sequence. (Positive values of \( X \) correspond to money won and negative values to money lost.) Obviously the maximum value of \( X \) is 1200 (if she wins all 100 games) and the minimum is −300 (if she loses all 100 games). Compute the following:

(a) \( E(X) = \)
(b) $Var(X) =$